

Brans-Dicke DGP Brane Cosmology

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Abstract

We consider a five dimensional DGP-brane scenario endowed with a non-minimally coupled scalar field within the context of Brans-Dicke theory. This theory predicts that the mass appearing in the gravitational potential is modified by the addition of the mass of the effective intrinsic curvature on the brane. We also derive the effective four dimensional field equations on a $3 + 1$ dimensional brane where the fifth dimension is assumed to have an orbifold symmetry. Finally, we discuss the cosmological implications of this setup, predicting an accelerated expanding universe with a value of the Brans-Dicke parameter ω consistent with values resulting from the solar system observations.

1 Introduction

Over the past decade the possibility of the observable Universe being a brane-world [1] embedded in a higher dimensional space-time has generated a great amount of interest. This was motivated by the fact that there is a strongly coupled sector of $E_8 \times E_8$ heterotic string theory which can be described by a field theory living in 11-dimensional space-time [2]. The 11-dimensional world consists of two 10-dimensional hypersurfaces embedded on the fixed points of an orbifold and the matter fields are assumed to be confined and live on these hypersurfaces which are known to be 9-branes. After compactification of the 11-dimensional theory on a Calabi-Yau 3-fold, we obtain an effective 5-dimensional theory [3] which has the structure of two 3-branes located on the orbifold boundaries. This scenario has motivated intense efforts to understand the case where the bulk is a 5-dimensional anti de-Sitter space. In this setup, gravitons are allowed to penetrate into the bulk but are localized on and around the brane [4]. It was then shown that in a background of a non-factorizable geometry an exponential warp factor emerges which multiplies the Poincaré invariant 3+1 dimensions in the metric. The model consists of two 3-branes situated along the 5th dimension, compactified on a S^1/Z_2 orbifold symmetry where the two branes must have opposite tensions. The evolution equation followed from such a brane scenario differs from that of the standard four dimensional evolution equation when no branes are present [6]. The existence of branes and the requirement that matter fields should be localized on the brane lead to a non-conventional cosmology, necessitating a more concerted study. A large number of studies have been devoted to the effective gravity induced on the brane [7] and, in particular, a great amount of interest was generated on inflationary cosmology [8]. More recently, post-inflationary brane cosmology has been also considered in [9]. Not surprisingly, the problem of the cosmological constant has become a focal point in the brane-world studies where, for example, in [10, 11, 12] a five dimensional action with a scalar field is non-minimally coupled to five dimensional gravity and to the four dimensional brane tension. There has also been some discussion on the localization of gravity [13]. A feature common to these type of models is that they predict deviations from the usual $4D$ gravity at short distances.

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A somewhat different approach within the brane-world framework is the model proposed by Dvali, Gabadadze and Porrati (DGP) [14, 15]. It predicts deviations from the standard $4D$ gravity over large distances. The transition between four and higher-dimensional gravitational potentials in the DGP model arises because of the presence of both the brane and bulk Einstein terms in the action. The Friedmann-like equations governing the cosmological evolution of a brane possessing an intrinsic curvature term in its action have already been derived and discussed for an AdS-Schwarzschild bulk space-time [16]. Cosmological consideration of the DGP model was first discussed in [17] where it was shown that in a Minkowski bulk space time we can obtain self-accelerating solutions. In the original DGP model it is known that $4D$ general relativity is not recovered at linearized level. However, some authors have shown that at short distances we can recover the $4D$ general relativity in a spherically symmetric configuration, see for example [18]. An important observation was made in [19, 20] where it was shown that the DGP model allows for an embedding of the standard Friedmann cosmology whereby the cosmological evolution of the background metric on the brane can entirely be described by the standard Friedmann equation plus energy conservation on the brane. This was later extended to arbitrary number of transverse dimensions in [21]. For a comprehensive review of the phenomenology of DGP cosmology, the reader is referred to [22].

It is worth mentioning that an interesting feature of the original DGP model is the existence of ghost-like excitations [23]. We do not endeavor to discuss such ghost excitation in the present study since our aim is the study of the cosmological implications of the model presented in this work. For a comprehensive review of the existence of $4D$ ghosts on the self-accelerating branch of solutions in DGP models see [24].

In this paper we consider the dynamics of a scalar field existing in the bulk and study the cosmological implications that such a scenario would entail in a DGP brane-world. We do this by studying the effective field equations on the $3 + 1$ dimensional DGP brane which is assumed to be rigidly located on the orbifold symmetry along the 5th dimension. In section 3 we consider the weak field approximation and show that this theory predicts a modification to the mass appearing in the gravitational potential in the form of an additional mass term related to the effective intrinsic curvature and that such a mass can be interpreted as dark matter. Finally, in section 4, we discuss the ensuing cosmology, showing that the model predicts an accelerated expanding universe without introducing dark energy and is consistent with the present observational bounds on the value of the Brans-Dicke parameter ω . Conclusions are drawn in the last section.

2 Brans-Dicke Brane in DGP Scenario

Let us start by considering a bulk scalar field non-minimally coupled to gravity in a 5-dimensional DGP brane world. Such a scalar field in the five dimensional theory can be viewed as a dilaton which is purely an outcome of dimensional reduction from some higher dimensional theory to a 5-dimensional space-time [25]. The action for our model can be written as

$$S_5 = \frac{1}{2\kappa_{(5)}^2} \int d^5x \sqrt{-g} \left(\phi \mathcal{R} - \frac{\omega}{\phi} \partial_A \phi \partial^A \phi \right) + \frac{1}{2\mu^2} \int d^4x \sqrt{-q} R \Big|_{\text{brane}} + \int d^5x \sqrt{-g} \mathcal{L}_m, \quad (1)$$

where \mathcal{R} is the Ricci scalar associated with the 5-dimensional space-time metric g_{AB} , ϕ is a scalar field which we shall call the Brans-Dicke (BD) field, ω is a dimensionless coupling constant which determines the coupling between gravity and the BD scalar field. Similarly, the second term is the Einstein-Hilbert action for the induced metric $q_{\mu\nu}$ on the brane where R is the relevant scalar curvature, $\mu^2 = 8\pi G_{(4)}$ and \mathcal{L}_m represents the Lagrangian for the matter fields. Latin indices denote 5-dimensional components ($A, B = 0, \dots, 5$) and for convenience we choose $\kappa_{(5)}^2 = 8\pi G_{(5)} = 1$. The induced metric $q_{\mu\nu}$ is defined as usual from the bulk metric g_{AB} by

$$q_{\mu\nu} = \delta_\mu^A \delta_\nu^B g_{AB}. \quad (2)$$

The variational derivative of the action equation (1) with respect to g_{AB} yields the field equations

$$G_{AB} \equiv \mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R} = \frac{1}{\phi} \left[T_{AB}^\phi + T_{AB}^{curv} + T_{AB} \right], \quad (3)$$

where

$$T_{AB}^\phi = \frac{\omega}{\phi} \left[\phi_{;A}\phi_{;B} - \frac{1}{2}g_{AB}\phi_{;C}\phi^{;C} \right] + [\phi_{;AB} - g_{AB}\phi^{;C}_{;C}], \quad (4)$$

and

$$T_{AB}^{curv} = -\frac{1}{\mu^2}g_A^\mu g_B^\nu \left[R_{\mu\nu} - \frac{1}{2}q_{\mu\nu}R \right]. \quad (5)$$

Note that T_{AB}^{curv} is the contribution coming from the scalar curvature of the brane. The equation of motion for the scalar field ϕ is given by

$$\square\phi = \frac{(T + T^{curv})}{3\omega + 4}, \quad (6)$$

where $T = T^C_C$ is the trace of the energy-momentum tensor of the matter content of the 5-dimensional space-time. Notice the factor $3\omega + 4$ in the denominator on the right hand side of the BD field equation instead of the familiar $2\omega + 3$ in the 4-dimensional case [26]. This is determined by requiring the validity of the equivalence principle in our setup, see [27] for a discussion of this topic in the context of 4-dimensional Brans-Dicke theory.

3 Brans-Dicke DGP brane model in the weak field approximation regime

Although equations (3) and (6) represent the more usual or standard form of Brans-Dicke equations in the DGP model but to consider the weak field approximation of the model we shall work in the Einstein frame [28, 29] given by

$$\tilde{G}_{AB} = \mathcal{G} \left[\tilde{T}_{AB} + \frac{3\omega + 4}{2\mathcal{G}\phi^2} \left(\phi_{;A}\phi_{;B} - \frac{1}{2}\tilde{g}_{AB}\phi_{;C}\phi^{;C} \right) + \tilde{T}_{AB}^{curv} \right], \quad (7)$$

$$\tilde{\square}\ln(\mathcal{G}\phi) = \frac{\mathcal{G}}{3\omega + 4}[\tilde{T} + \tilde{T}^{curv}], \quad (8)$$

which is obtained from (3) and (6) by making the transformation

$$\tilde{g}_{AB} = \mathcal{G}\phi g_{AB}, \quad (9)$$

$$\tilde{T}_{AB} = \mathcal{G}^{-1}\phi^{-1}T_{AB} \quad \text{and} \quad \tilde{T}_{AB}^{curv} = \mathcal{G}^{-1}\phi^{-1}T_{AB}^{curv}, \quad (10)$$

where \mathcal{G} is an arbitrary constant and the tilde on \tilde{G}_{AB} , $\tilde{\square}$, \tilde{T}^{curv} and \tilde{T} means that these quantities are calculated using the conformal metric \tilde{g}_{AB} .

In the weak field approximation of Brans-Dicke theory, in addition to

$$g_{AB} = \eta_{AB} + h_{AB}, \quad (11)$$

we must also assume that

$$\phi = \phi^{(0)} + \phi^{(1)}, \quad (12)$$

where $\phi^{(1)} = \phi^{(1)}(x^\mu, y)$ is a first-order term in the energy density and $\left| \frac{\phi^{(1)}}{\phi^{(0)}} \right| \ll 1$. Taking into account (12) and setting $\mathcal{G} = \frac{1}{\phi^{(0)}}$ the transformation equations (9) and (10) become

$$\tilde{g}_{AB} = \eta_{AB} + \tilde{h}_{AB}, \quad (13)$$

$$\tilde{T}_{AB} = (1 - \phi^{(1)}\mathcal{G})T_{AB} = T_{AB} \quad (14)$$

and

$$\tilde{T}_{AB}^{curv} = (1 - \phi^{(1)}\mathcal{G})T_{AB}^{curv} = T_{AB}^{curv}, \quad (15)$$

where

$$\tilde{h}_{AB} = h_{AB} + \phi^{(1)}\mathcal{G}\eta_{AB}, \quad (16)$$

and only the first-order terms in the mass and curvature densities have been retained. Note that T_{AB}^{curv} lives on the brane. Now, substituting (12) in the field equations (7) and taking equations (13) and (14) into account, we get

$$\tilde{G}_{AB} = \mathcal{G}[T_{AB} + T_{AB}^{curv}]. \quad (17)$$

On the other hand, the scalar field equation (6) becomes

$$\square\phi^{(1)} = \frac{[T + T^{curv}]}{3\omega + 4}. \quad (18)$$

It turns out then that equations (17) are formally identical to the field equations of General Relativity with \mathcal{G} replacing the Newtonian gravitational constant $G_{(5)}$, dropping the coefficient 8π . Therefore, if $\tilde{g}_{AB}(G_{(5)}, x^\mu, y)$ is a known solution of the Einstein equations in the weak field approximation for a given T_{AB} , then the Brans-Dicke solution corresponding to the same T_{AB} will be given in the weak field approximation just by taking the inverse of equation (9), that is

$$g_{AB}(x^\mu, y) = \mathcal{G}^{-1}\phi^{-1}\tilde{g}_{AB}(\mathcal{G}, x^\mu, y) = \left[1 - \phi^{(1)}(x^\mu, y)\mathcal{G}\right]\tilde{g}_{AB}(\mathcal{G}, x^\mu, y), \quad (19)$$

or, equivalently,

$$h_{AB}(x^\mu, y) = \tilde{h}_{AB}(\mathcal{G}, x^\mu, y) - \phi^{(1)}(x^\mu, y)\mathcal{G}\eta_{AB}. \quad (20)$$

We therefore conclude that the general problem of finding solutions of Brans-Dicke DGP equations of gravity in the weak field approximation may be reduced to solving Einstein field equations for the same matter distribution. It should be noted that the Einstein tensor \tilde{G}_{AB} which appears on the left hand side of (17) must be calculated in the weak field approximation, i.e., taking \tilde{g}_{AB} as given by (13).

Now, it is well known from equation (17) that the gravitational potential in the weak field approximation of DGP expressed in Gaussian normal coordinates in the Einstein frame is given by [20]

$$\tilde{U}(\vec{r}) = -\frac{\mu^2 M}{6\pi r} \left[\cos(2\mu^2 r) - \frac{2}{\pi} \cos(2\mu^2 r) \text{si}(2\mu^2 r) + \frac{2}{\pi} \sin(2\mu^2 r) \text{ci}(2\mu^2 r) \right], \quad (21)$$

where the sine and cosine integrals are defined by the following relations.

$$\text{si}(x) = \int_0^x d\xi \frac{\sin \xi}{\xi} \quad \text{and} \quad \text{ci}(x) = -\int_x^\infty d\xi \frac{\cos \xi}{\xi}. \quad (22)$$

Note that this equation is the gravitational potential for the mass density $\rho(\vec{r}) = M\delta(\vec{r})$. We must now solve equation (18), leading to $\phi^{(1)}$ given by

$$\phi^{(1)} = -\frac{M + M^{curv}}{4\pi(3\omega + 4)r}. \quad (23)$$

Using equations (16), (21) and (23) gravitational potential in Jordan frame given by

$$U(\vec{r}) = \tilde{U}(\vec{r}) - \frac{\mathcal{G}(M + M^{curv})}{4\pi(3\omega + 4)r}, \quad (24)$$

where it seems as if M^{curv} plays the role of dark matter which has a contribution to the gravitational potential. It therefore seems plausible that $\rho^{curv}(\vec{r}) = M^{curv}\delta(\vec{r})$ could be considered as a candidate for dark matter. From equation (24) we see that the Brans-Dicke DGP model predicts a transition scale relating the 4 and 5-dimensional behavior of the gravitational potential in the Jordan frame

$$\begin{cases} r \ll \ell_{DGP} : U(\vec{r}) = -\frac{4\mu^2 M(3\omega+4)+6\mathcal{G}(M+M^{curv})}{24\pi(3\omega+4)r} - \frac{\mu^2 M}{6\pi r} \left[\left(\gamma - \frac{2}{\pi}\right) \frac{r}{\ell_{DGP}} + \frac{r}{\ell_{DGP}} \ln\left(\frac{r}{\ell_{DGP}}\right) + \mathcal{O}\left(\frac{r}{\ell_{DGP}}\right)^2 \right], \\ r \gg \ell_{DGP} : U(\vec{r}) = -\frac{\mathcal{G}(M+M^{curv})}{4\pi(3\omega+4)r} - \frac{\mu^2 M}{6\pi^2 r^2} - \frac{\mu^2 M}{6\pi^2 r^2} \left[-2\frac{\ell_{DGP}^2}{r^2} + \mathcal{O}\left(\frac{\ell_{DGP}}{r}\right)^4 \right]. \end{cases}$$

Here $\ell_{DGP} = \frac{\mu^2}{2}$ and $\gamma \simeq 0.577$ is Euler's constant.

4 Cosmological considerations

Before we discuss the energy-momentum tensor, let us define the five dimensional metric which has the following form

$$ds^2 = g_{AB}dx^A dx^B = q_{\mu\nu}(x^\mu, y) dx^\mu dx^\nu + b^2(x^\mu, y) dy^2, \quad (25)$$

where $\mu, \nu = 0, \dots, 3$ and y is the coordinate associated with the fifth dimension and we will adopt a brane-based approach where the brane is the hypersurface defined by $y = 0$. We also assume an orbifold symmetry along the fifth direction $y \rightarrow -y$. As we shall see in the coming sections, this will help us to simplify our calculations. Next we define the energy-momentum tensor

$$T^A_B = T^A_B|_{\text{bulk}} + T^A_B|_{\text{brane}}, \quad (26)$$

where the subscripts “brane” and “bulk” refer to the corresponding energy-momentum tensors. For simplicity we assume that the bulk is devoid of matter other than the Brans-Dicke scalar field. The brane matter field is held at $y = 0$ with the following energy momentum tensor

$$T^A_B|_{\text{brane}} = \frac{\delta(y)}{b} \text{diag}(-\rho, p, p, p, 0) \quad \text{and} \quad T^A_B|_{\text{bulk}} = \text{diag}(0, 0, 0, 0, 0). \quad (27)$$

The above expressions are written assuming that the brane is thin and that the bulk is empty.

Since we are interested in exploring the spatially flat cosmology ($k = 0$), we consider a 5-dimensional flat metric *ansatz* of the following form

$$ds^2 = -n^2(\tau, y) d\tau^2 + a^2(\tau, y) \delta_{ij} dx^i dx^j + b^2(\tau, y) dy^2, \quad (28)$$

where $i, j = 1, 2, 3$. With this metric we are now able to write the equations of motion. The (0,0) component reads

$$3 \left[\frac{\dot{a}}{a} \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left(\frac{a''}{a} + \frac{a'}{a} \left(\frac{a'}{a} - \frac{b'}{b} \right) \right) \right] = \frac{1}{\phi} [T_{00}^\phi + T_{00} + T_{00}^{curv}], \quad (29)$$

where

$$T_{00}^{\phi} = -\dot{\phi} \left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b} - \frac{\omega}{2} \frac{\dot{\phi}}{\phi} \right) + \left(\frac{n}{b} \right)^2 \left[\phi'' + \phi' \left(3\frac{a'}{a} - \frac{b'}{b} + \frac{\omega}{2} \frac{\phi'}{\phi} \right) \right] \quad (30)$$

and

$$T_{00}^{curv} = -\frac{3}{\mu^2 b} \left(\frac{\dot{a}}{a} \right)^2 \delta(y). \quad (31)$$

The (i, j) components are given by

$$\left\{ -2\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \left[\frac{\dot{a}}{a} \left(-\frac{\dot{a}}{a} + 2\frac{\dot{n}}{n} \right) + \frac{\dot{b}}{b} \left(-2\frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) \right] \right\} \delta_{ij} + \left\{ \left(\frac{n}{b} \right)^2 \left[2\frac{a''}{a} + \frac{n''}{n} + \frac{a'}{a} \left(\frac{a'}{a} + 2\frac{n'}{n} \right) - \frac{b'}{b} \left(\frac{n'}{n} + 2\frac{a'}{a} \right) \right] \right\} \delta_{ij} = \frac{1}{\phi} \left(\frac{n}{a} \right)^2 [T_{ij}^{\phi} + T_{ij} + T_{ij}^{curv}], \quad (32)$$

where

$$T_{ij}^{\phi} = \left\{ \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} \left(2\frac{\dot{a}}{a} + \frac{\dot{b}}{b} - \frac{\dot{n}}{n} + \frac{\omega}{2} \frac{\dot{\phi}}{\phi} \right) - \left(\frac{n}{b} \right)^2 \left[\frac{\phi''}{\phi} + \frac{\phi'}{\phi} \left(2\frac{a'}{a} \frac{b'}{b} + \frac{n'}{n} + \frac{\omega}{2} \frac{\phi'}{\phi} \right) \right] \right\} \delta_{ij} \quad (33)$$

and

$$T_{ij}^{curv} = -\frac{1}{\mu^2 b} \left[\frac{a^2}{n^2} \delta_{ij} \left(-\frac{\dot{a}^2}{a^2} + \frac{\dot{a}}{a} \frac{\dot{n}}{n} - 2\frac{\ddot{a}}{a} \right) \right] \delta(y). \quad (34)$$

The $(0, 5)$ component takes the form

$$3 \left(\frac{\dot{a}}{a} \frac{n'}{n} + \frac{\dot{b}}{b} \frac{a'}{a} - \frac{\dot{a}'}{a} \right) = \frac{1}{\phi} T_{05}^{\phi}, \quad (35)$$

where

$$T_{05}^{\phi} = \dot{\phi}' - \dot{\phi} \left(\frac{n'}{n} - \omega \frac{\phi'}{\phi} \right) - \frac{\dot{b}}{b} \phi'. \quad (36)$$

Finally, for the $(5, 5)$ component one has

$$3 \left[-\left(\frac{\ddot{a}}{a} + \frac{\dot{a}}{a} \left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \right) + \left(\frac{n}{b} \right)^2 \left(\frac{a'}{a} \left(\frac{a'}{a} + \frac{n'}{n} \right) \right) \right] = \frac{1}{\phi} \left(\frac{n}{b} \right)^2 [T_{55}^{\phi} + T_{55}], \quad (37)$$

where

$$T_{55}^{\phi} = \ddot{\phi} + \dot{\phi} \left(3\frac{\dot{a}}{a} - \frac{\dot{n}}{n} + \frac{\omega}{2} \frac{\dot{\phi}}{\phi} \right) - \left(\frac{n}{b} \right)^2 \phi' \left(3\frac{a'}{a} + \frac{n'}{n} - \frac{\omega}{2} \frac{\phi'}{\phi} \right). \quad (38)$$

The equation of motion for the Brans-Dicke field reads

$$\ddot{\phi} + \dot{\phi} \left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b} - \frac{\dot{n}}{n} \right) - \left(\frac{n}{b} \right)^2 \left[\phi'' + \phi' \left(3\frac{a'}{a} - \frac{b'}{b} + \frac{n'}{n} \right) \right] = -n^2 \frac{(T + T^{curv})}{3\omega + 4}, \quad (39)$$

where a dot represents the time derivative with respect to τ and the prime corresponds to derivatives with respect to y . Note that in the above derivation we have assumed $\phi = \phi(\tau, y)$. We make the assumption that the metric and the BD field are continuous across the brane localized at $y = 0$. However, their derivatives can be discontinuous at the brane position in the y direction. This suggests the second derivatives of the scale factor and the BD field will have a Dirac delta function associated with the position of the brane. Since the matter is localized on the brane it will introduce a delta

function in the Einstein equations which will be matched by the distributional part of the second derivatives of the scale factor and the BD field.

Using the Einstein equations it is possible to find out the jump conditions for a and n by matching the Dirac delta functions appearing on the left-hand side of the Einstein equations to the ones coming from the energy-momentum tensor equation (26). For the BD field one has to use equation (39) to evaluate the jump conditions. We therefore find

$$\frac{[a']_0}{a_0 b_0} = -\frac{1}{(3\omega + 4)\phi_0} \left[p + p^{curv} + (\omega + 1)(\rho + \rho^{curv}) \right], \quad (40)$$

$$\frac{[n']_0}{n_0 b_0} = \frac{1}{(3\omega + 4)\phi_0} \left[3(\omega + 1)(p + p^{curv}) + (2\omega + 3)(\rho + \rho^{curv}) \right], \quad (41)$$

$$\frac{[\phi']_0}{\phi_0 b_0} = \frac{2}{(3\omega + 4)\phi_0} \left[\gamma\rho + \gamma_c \rho^{curv} \right], \quad (42)$$

where

$$\rho^{curv} = -\frac{3}{\mu^2 n_0^2} \left(\frac{\dot{a}_0}{a_0} \right)^2, \quad (43)$$

$$p^{curv} = \frac{1}{\mu^2 n_0^2} \left(\frac{\dot{a}_0^2}{a_0^2} - 2 \frac{\dot{a}_0}{a_0} \frac{\dot{n}_0}{n_0} + 2 \frac{\ddot{a}_0}{a_0} \right), \quad (44)$$

$$\gamma = \frac{1}{2}(3w_m - 1) \quad \text{and} \quad \gamma_c = \frac{1}{2}(3w_c - 1), \quad (45)$$

with $w_c = \frac{p^{curv}}{\rho^{curv}}$ and the subscript 0 stands for the brane at $y = 0$. The first two conditions, equations (40) and (41), are equivalent to Israel's junction conditions in general relativity [30] (see [6] for a discussion of its application in the context of brane world). It is important to note that the above jump conditions at $y = 0$ depend on the energy density and the pressure component of the brane world and induced curvature on the brane. Interestingly, for the radiation dominated phase on the brane, $\rho = 3p$, the jump condition for ϕ does not vanish and is proportional to the energy density and pressure component of the induced curvature on the brane.

Taking the jump of the (0,5) component of the Einstein equation and substituting equations (40 and 41) we get the continuity equation for the matter on the brane

$$\dot{\rho} + 3(\rho + p) \frac{\dot{a}}{a} = 0. \quad (46)$$

Equation (46) shows that the energy content of the brane is still conserved in this scenario which seems to be at odds with recent results obtained independently by several authors who conclude that the presence of a dilaton field in the bulk will lead to a non-trivial coupling with the matter on the brane which, from the point of view of an observer living on the brane, would be seen as matter leaking from the brane [12, 31, 32, 33, 34]. Our situation is different because the coupling between the BD field and ordinary matter on the brane was chosen so as to satisfy the equivalence principle, as was mentioned above. Had we chosen the coupling in (39) to have the usual 4-dimensional value $(2\omega + 3)^{-1}$, we would have ended up with a situation where the conservation equation (46) would not hold and energy could leak from the brane.

Taking the mean value of the (5,5) component of Einstein's equations we can now obtain a Friedmann type equation on the brane by following a very similar procedure to the one introduced in [6]. Using the fact that due to the orbifold symmetry $y \leftrightarrow -y$, the mean value of any of the quantities a , n or ϕ should be zero, we can discard all the terms involving mean values in the average of the (5,5) component of the Einstein equations. The equation so obtained will still involve a term containing $\ddot{\phi}$, but we can use the mean value of the BD field equation (39) to write this in terms of a ,

b and n , and their derivatives. After a somewhat lengthy calculation we obtain the first Friedmann type equation on the brane

$$\begin{aligned} \frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{\omega}{6} \left(\frac{\dot{\phi}_0}{\phi_0}\right)^2 + \frac{2\gamma\omega}{\mu^2(3\omega+4)} \frac{\dot{a}_0^2}{a_0^2\phi_0^2} - \frac{1}{2\mu^2(3\omega+4)} \frac{\dot{a}_0^2}{a_0^2\phi_0^2} \left[p + \rho + \frac{\omega}{2}(3p + \rho)\right] \\ + \frac{1}{2} \frac{\dot{a}_0^4}{a_0^4\phi_0^2} \left[\frac{6\omega}{(3\omega+4)} + 1\right] + \frac{2\gamma\omega}{\mu^2(3\omega+4)^2} \frac{\ddot{a}_0}{a_0\phi_0^2} + \frac{1}{2\mu^2(3\omega+4)^2} \frac{\ddot{a}_0}{a_0\phi_0^2} (p + \rho + \omega\rho) + \\ \frac{1}{2\mu^4(3\omega+4)} \frac{\dot{a}_0^2\ddot{a}_0}{a_0^3\phi_0^2} \left[\frac{12\omega}{(3\omega+4)} - 2 - 3\omega\right] + \frac{1}{2\mu^4(3\omega+4)} \frac{\ddot{a}_0^2}{a_0^2\phi_0^2} \left[\frac{6\omega}{(3\omega+4)} + 1\right] = -\frac{\omega}{4(3\omega+4)^2\phi_0^2} \\ \times \left[3p^2 + 2p\rho + \frac{\rho^2}{3}\right] - \frac{1}{4(3\omega+4)\phi_0^2} \left[\frac{p^2}{2} + p\rho(1+\omega) + \frac{\rho^2}{6}(3+2\omega)\right]. \end{aligned} \quad (47)$$

Upon using the components (5, 5) and (0, 5) we obtain the other Friedmann type equation

$$\begin{aligned} \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{1}{(3\omega+4)^2\phi_0^2} \left[p^{tot} + (\omega+1)\rho^{tot}\right]^2 - \frac{2}{3a_0^4} \int \frac{d\tau}{\phi_0} \dot{a}_0 a_0^3 T^{\phi 5}_5 \Big|_{y=0} \\ - \frac{2}{3a_0^4(3\omega+4)} \int \frac{d\tau}{\phi_0^2} \left[p^{tot} + (\omega+1)\rho^{tot}\right] a_0^4 T^{\phi 5}_0 \Big|_{y=0}. \end{aligned} \quad (48)$$

While deriving the above equations we have assumed that from the point of view of the brane observer, the extra dimension is static, that is $b = b_0$. We have also fixed the time in such a way that $n_0 = 1$, corresponding to the usual choice of time in conventional cosmology. It is interesting to note that by taking the limit $\omega \rightarrow \infty$ on the right-hand side of equation (47), we obtain exactly the same expression as in the DGP model [20]. Note that in this model ω is a coupling constant, but after inducing equations on the brane it can be interpreted as the induced BD parameter on the brane.

Taking the mean value of the BD field equation we obtain an equation of motion for ϕ on the brane

$$\begin{aligned} \frac{\ddot{\phi}_0}{\phi_0} + 3 \frac{\dot{a}_0}{a_0} \frac{\dot{\phi}_0}{\phi_0} + \frac{3\omega}{\mu^2(3\omega+4)^2\phi_0^2} \frac{\ddot{a}_0}{a_0} \left[2\gamma + \frac{6}{\mu^2} \frac{\dot{a}_0^2}{a_0^2} + \frac{3}{\mu^2} \frac{\ddot{a}_0}{a_0}\right] \\ + \frac{3\omega}{\mu^2(3\omega+4)^2\phi_0^2} \frac{\dot{a}_0^2}{a_0^2} \left[2\gamma + \frac{3}{\mu^2} \frac{\dot{a}_0^2}{a_0^2}\right] = \frac{\omega\gamma^2}{(3\omega+4)^2\phi_0^2}. \end{aligned} \quad (49)$$

Note that in order to obtain equation (49) we also have to assume that the non-distributional part of ϕ'' vanishes, otherwise, a term involving $\widehat{\phi''}$ will appear in the BD field equation. As we shall see in the next section it is possible to obtain cosmologically interesting solutions which verify this condition. Using equations (47), (48) and (49) we can determine the cosmology of the brane at $y = 0$.

5 Cosmological solutions

It is well known that the original DGP brane cosmology model leads to self accelerating solutions [17]. In this section, we start by consider the self accelerating solutions in our model in the vacuum sector *i.e* $\rho = p = 0$. Let us take the brane metric, $q_{\mu\nu}$, as spatially flat in the $4D$ de Sitter geometry and write

$$q_{\mu\nu} dx^\mu dx^\nu = -d\tau^2 + e^{2H\tau} d\mathbf{x}^2. \quad (50)$$

Thus, in order to obtain the self accelerating solutions, we make the *ansatz*

$$\phi_0(\tau) = \alpha a_0(\tau), \quad (51)$$

where α is an arbitrary positive constant. Using equations (48), (50) and (51) one finds

$$H = \epsilon \mu^2 (3\omega + 4) \phi_0 \sqrt{\frac{\omega + 20}{6\omega}}, \quad (52)$$

where $\epsilon = \pm 1$. Note that for $\epsilon = +1$, or the self-accelerating branch, we will have the self accelerating solutions in our model.

We now obtain solutions for a and ϕ which will allow us to discuss the cosmological implications in our setup. Let us then assume that both the total energy density ρ and pressure p on the brane consist of two parts

$$\rho = \lambda + \varrho \quad \text{and} \quad p = -\lambda + \mathbf{p}, \quad (53)$$

where λ , ϱ and \mathbf{p} are the tension, the usual cosmological energy density and pressure in the matter frame respectively. In what follows we concentrate on the case $\varrho = \mathbf{p} = 0$, *i.e.* the vacuum solution. However, by retaining a non-zero effective tension on the brane we are actually taking the brane effects into account. For simplicity we take the form of the tension, λ , as follows [35]

$$\lambda = \lambda_c \phi^2, \quad (54)$$

where λ_c is a constant. As before, we assume that the extra space-like dimension is stabilized (b is constant) and choose time such that $n_0 = 1$. We now look for a power law solution for the scale factor. Substituting the *ansätze*

$$a_0(\tau) \propto \tau^n \quad \text{and} \quad \phi_0(\tau) = \frac{\tau^m}{\mu^2}, \quad (55)$$

into equations (47) and (49) one obtains

$$m = -1, \quad (56)$$

and from equation (49), n is obtained as the solution of the following algebraic equation in terms of λ_c , μ^2 and ω

$$\begin{aligned} \frac{36\omega}{(3\omega + 4)^2} n^4 - \frac{36\omega}{(3\omega + 4)^2} n^3 + n^2 \left[\frac{9\omega}{(3\omega + 4)^2} - \frac{24\lambda_c \omega}{\mu^2 (3\omega + 4)^2} \right] \\ - n \left[3 + \frac{12\lambda_c \omega}{\mu^2 (3\omega + 4)^2} \right] + \frac{4\lambda_c^2 \omega}{\mu^4 (3\omega + 4)^2} + 2 = 0. \end{aligned} \quad (57)$$

This algebraic equation has four explicit real solutions for n in terms of λ_c , μ^2 and ω .

Equation (55) shows that there is a possibility of having an accelerated expanding universe for some choices of ω , μ^2 and λ_c . The deceleration parameter on the brane as a function of ω , μ^2 and λ_c is given by

$$q(\omega, \lambda_c, \mu^2) = -\frac{a_0 \ddot{a}_0}{\dot{a}_0^2} = -\frac{n-1}{n}, \quad (58)$$

where n is one of the four roots of equation (57). A glance at equation (58) reveals that using the condition for acceleration, that is $q(\omega, \lambda_c, \mu^2) < 0$, leads to $n > 1$ from which, using definition $w_{\text{eff}} = -1 - \frac{2\dot{H}_0}{3H_0^2}$ for the effective quintessence, we find $w_{\text{eff}} < -1/3$. Figure 1 shows the behavior of the deceleration parameter, q , as a function of ω for $\frac{\lambda_c}{\mu^2} \sim 1$ and n , taken as a root of equation (57). Therefore, each graph in this figure corresponds to one of the roots of the fourth order algebraic equation (57). As can be seen, the graphs in the first row in Figure 1 show that for positive and negative values of ω , we have an accelerating universe. The graph on the top left hand corner is particularly interesting since it shows that for $\omega \rightarrow \pm\infty$ we have $q \rightarrow -1$, that is the universe finally approaches the eternal de Sitter phase. It has been claimed that the value of $|\omega|$ in $4D$ should be large (> 40000) if Brans-Dicke theory is to be consistent with the astronomical observations [36].

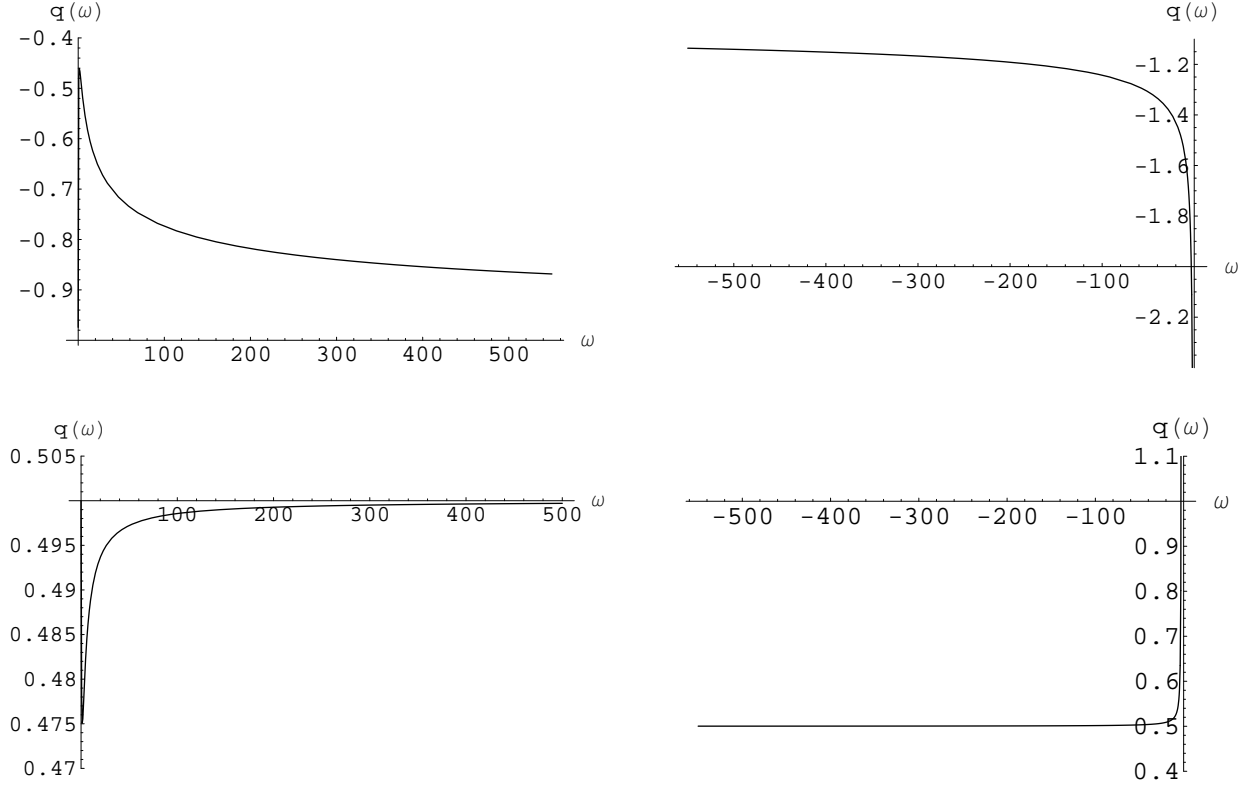


Figure 1: The behavior of $q(\omega)$ as a function of ω for $\frac{\lambda_S}{\mu^2} \sim 1$. Each figure represents the deceleration parameter for one of the four roots of equation (57).

Therefore, the graph mentioned above seems to be in good agreement with the observational data. The graphs in the second row in figure 1 predict a decelerating universe which is not consistent with the present observational data. However, the model can be used to interpolate back the deceleration parameter to an earlier epoch to yield a decelerating universe which is required in order to explain processes like nucleosynthesis.

6 Conclusions

We have derived the effective four dimensional field equations in a DGP brane-world setting where a BD field is assumed to be present in the bulk. We considered the weak field approximation in our model which resulted in the modification of the mass appearing in the gravitational potential by the addition of the mass of the effective intrinsic curvature that appears on the brane. In our scenario, the conservation equation for the matter fields confined to the brane still holds in spite of the existence of a BD field in the bulk. This is due to the fact that our calculations were done in the Jordan frame. Finally we obtained the modified Friedmann equation on the brane and showed that an accelerated expanding universe results for certain choices of the parameters which is in good agreement with the present bounds on the value of ω in Brans-Dicke theory.

References

- [1] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, (1983) 136;
K. Akama, "Pregeometry" in Lecture Notes in Physics, 176, *Gauge Theory and Gravitation, Proceedings, Nara, 1982*, (Springer-Verlag), edited by K. Kikkawa, N. Nakanishi and H.

- Nariai, 267;
 Keiichi Akama, Lect. Notes Phys. **176** (1982) 267, hep-th/0001113.
- [2] P. Horava and E. Witten, Nucl. Phys. B **460** (1996) 506, hep-th/9510209 ;
 P. Horava and E. Witten, Nucl. Phys. B **475** (1996) 94, hep-th/9603142.
- [3] A. Lukas, B. A. Ovrut, K. Stelle and D. Waldram, Phys. Rev. D **60** (1999) 086001;
 J. E. Ellis, Z. Lalak, S. Pokorski and W. Pokorski, Nucl. Phys. B **540**(1999) 149.
- [4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**,(1999) 3370.
- [5] A. Mazumdar and J. Wang, Phys. Lett. B **490** (2000) 251.
- [6] P. Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B **565** (2000) 269;
 P. Binetruy, C. Deffayet, U. Ellwanger and D. langlois, Phys. Lett. B **477** (2000) 269;
 J. M. Cline, C. Grojean and G. Servant, Phys. Rev. Lett. **83** (1999) 4245;
 T. Shiromizu, K. Maeda and M. Sasaki, Phys. Rev. D **62** (2000) 024012, gr-qc/9910076.
- [7] N. Kaloper, Phys. Rev. D **60** (1999) 123506;
 M. Cevetic and Jing Wang, Phys. Rev. D **61** (2000) 124020;
 R. Maartens, Phys. Rev. D **62** (2000) 084023, hep-th/0004166;
 C. van de Bruck, M. Dorca, R. Brandenberger and A. Lukas, Phys. Rev. D **62** (2000) 123515, hep-th/0005032;
 D. Langlois, Phys. Rev. D **62** (2000) 126012, hep-th/0005025;
 B. Grinstein, D. R. Nolte and W. Skiba, Phys. Rev. D **62** (2000) 086006, hep-th/ 0005001;
 K. Koyama and J. Soda, Phys. Rev. D **62** (2000) 123502, hep-th/0005239;
 L. Anchordoqui, C. Nunez, K. Olsen, JHEP **0010** (2000) 050, hep-th/0007064.
- [8] T. Nihei, Phys. Lett. B **465** (1999) 81;
 C. Csaki, M. Graesser, C. Kolda and J. Terning, Phys. Lett. B **462** (1999) 34;
 P. Kanti, I. I. Kogan, K. A. Olive and M. Pospelov, Phys. Lett. B **468** (1999) 31;
 P. Kanti, I. I. Kogan, K. A. Olive and M. Pospelov, Phys. Rev. D **61** (2000) 106004;
 P. Kraus, JHEP **9912** (1999) 011;
 H. B. Kim and H. D. Kim, Phys. Rev. D **61** (2000) 064003;
 H. B. Kim, Phys. Lett. B **478** (2000) 285;
 H. Stoica, S. H. Tye, I. Wasserman, Phys. Lett. B **482** (2000) 205;
 C. Csaki, M. Graesser, L. randall and J. Terning, Phys. Rev. D **62** (2000) 045015, hep-ph/9911406;
 R. Maartens, D. Wands, B. A. Bassett and I. Heard, Phys. Rev. D **62** (2000) 041301;
 R. N. Mohapatra, A. Perez-Lorenzana and C. A. de Sousa Pires, Int. J. Mod. Phys. A **16** (2001) 1431, hep-ph/0003328;
 L. E. Mendes and A. Liddle, Phys. Rev. D **62** (2000) 103511, astro-ph/0006020;
 L. Anchordoqui, K. Olsen, Mod. Phys. Lett. A **16** (2001) 1157, hep-th/0008102;
 E. J. Copeland, A. R. Liddle and J. Lidsey, Phys. Rev. D **64** (2001) 023509, astro-ph/0006421.
- [9] A. Mazumdar, Phys. Rev. D **64** (2001) 027304, hep-ph/0007269;
 A. Mazumdar, Nucl. Phys. B **597** (2001) 561, hep-ph/0008087.
- [10] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Phys. Lett. B **480** (2000) 193.
- [11] S. Kachru, M. Schluz and E. Silverstein, Phys. Rev. D **62** (2000) 045021, hep-th/0001206.
- [12] P. Binetruy, J. M. Cline and C. Grojean, Phys. Lett. B **489** (2000) 403, hep-th/0007029.
- [13] S. Nojiri and S. D. Odintsov, Phys. Lett. B **484** (2000) 119;
 C. Gomez, B. Janssen and P. J. Silva, JHEP **0004** (2000) 024;
 N. Alonso-Alberca, B. Janssen and P. J. Silva, JHEP **0004** (2000) 027.

- [14] G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B **485** (2000) 208, hep-th/0005016.
- [15] G. Dvali, G. Gabadadze, Phys. Rev. D **63** (2001) 065007, hep-th/0008054;
G. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, Phys. Rev. D **65** (2002) 024031, hep-th/0106058.
- [16] H. Collins and B. Holdom, Phys. Rev. D **62** (2000) 105009;
H. Collins and B. Holdom, Phys. Rev. D **62** (2000) 124008.
- [17] C. Deffayet, Phys. Lett. B **502** (2001) 199, hep-th/0010186.
- [18] T. Tanaka, Phys. Rev. D **69** (2004) 024001, gr-qc/0305031.
- [19] R. Dick, Class. Quant. Grav. **18** (2001) R1, hep-th/0105320.
- [20] R. Dick, Acta Phys. Pol. B **32** (2001) 3669, hep-th/0110162.
- [21] R. Cordero, A. Vilenkin, Phys. Rev. D **65** (2002) 083519, hep-th/0107175.
- [22] A. Lue, Phys. Rep. **423** (2006) 1, astro-ph/0510068.
- [23] M. A. Luty, M. Porrati and R. Rattazzi, JHEP **09** (2003) 029, hep-th/0303116;
A. Nicolis and R. Rattazzi, JHEP **06** (2004) 059, hep-th/0404159;
K. Koyama, Phys. Rev. D **72** 123511, hep-th/0503191;
K. Izumi, K. Koyama and T. Tanaka, JHEP **0704** (2007) 053, hep-th/0610282.
- [24] C. Charmousis, R. Gregory, N. Kaloper and A. Padilla, JHEP **0610** (2006) 066, hep-th/0604086;
R. Gregory, N. Kaloper, R. C. Myers and A. Padilla, arXiv:0707.2666.
- [25] L. E. Mendes and A. Mazumdar, Phys. Lett. B **501** (2001) 249.
- [26] J. D. Barrow and J. P. Mimoso, Phys. Rev. D **50** (1994) 3746.
- [27] S. Weinberg, “*Gravitation and cosmology: principles and applications of the general theory of relativity*,” John Wiley & Sons, 1972.
- [28] R. H. Dicke, Phys. Rev. **125** (1962) 2163.
- [29] A. Barros and C. Romero, Phys. Lett. A **245** (1998) 31, gr-qc/9712080.
- [30] W. Israel, Nuovo Cimento B (1966) 44.
- [31] K. Maeda and D. Wands, Phys. Rev. D **62** (2000) 124009, hep-th/0008188.
- [32] A. Mennim and R. Battye, Class. Quant. Grav. **18** (2001) 2171, hep-th/0008192.
- [33] C. van de Bruck, M. Dorca, C. J. A. P. Martins and M. Perry, Phys. Lett. B **495** (2000) 183, hep-th/0009056.
- [34] K. Atazadeh, M. Farhoudi and H. R. Sepangi, Phys. Lett. B **660** (2008) 275, gr-qc/0801.1398.
- [35] O. Bertolami and P. J. Martins, Phys. Rev. D **61** (2000) 064007, gr-qc/9910056.
- [36] B. Bertotti, L. Less and P. Tortora, Nature **425** (2003) 374.